

Energy-level pinning and the 0.7 spin state in one dimension: GaAs quantum wires studied using finite-bias spectroscopy

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We study the effects of electron-electron interactions on the energy levels of GaAs quantum wires using finite-bias spectroscopy. We probe the energy spectrum at zero magnetic field, and at crossings of opposite-spin-levels in high in-plane magnetic field, B . Our results constitute direct evidence that spin-up (higher energy) levels pin to the chemical potential, μ , as they populate. We also show that spin-up and spin-down levels abruptly rearrange at the crossing in a manner resembling the magnetic phase transitions predicted to occur at crossings of Landau levels. This rearranging and pinning of subbands provides a phenomenological explanation for the 0.7 structure, a 1D nanomagnetic state, and its high- B variants.

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I. INTRODUCTION

Whereas many collective electron phenomena in two- and zero-dimensional electron systems are well understood, such as the fractional quantum Hall effect and Kondo effect, this cannot be said of one-dimensional electron systems (1DES). As these may conceivably form the building-blocks of quantum circuits, it is important that their properties are understood. Theoretically, an interacting 1DES can be treated as a Luttinger-Liquid (LL) [1]; although tunnelling experiments in parallel semiconductor QWs [2] and carbon nanotubes [3] have shown evidence of Luttinger liquid behaviour, many QW characteristics cannot at present be understood within the Luttinger liquid framework. In particular, a spin-related phenomenon known as the *0.7 structure* [4, 5, 6] has long resisted quantitative explanation.

According to non-interacting electron theories, the conductance of a semiconductor 1DES is quantized at $N(2e^2/h)$, where N 1D modes lie below the Fermi energy. In real systems however, an additional plateau occurs at around $0.7 \times 2e^2/h$ — the 0.7 structure. This deceptively simple feature has attracted much experimental [4, 7, 8, 9, 10, 11, 12, 13, 14] and theoretical [15, 16, 17, 18, 19, 20, 21] interest, because its unusual magnetic field (B) and temperature (T) dependences [4, 10, 11, 12] imply that complex electron spin interactions strongly influence the behaviour of even the simplest quantum devices.

The 0.7 structure evolves continuously into the lowest energy spin-down mode with increasing B , implying that it is a type of spontaneously spin-polarised state. Whereas the 0.7 structure occurs at the $B = 0$ crossing of the $1\uparrow$ and $1\downarrow$ subbands, related conductance structures called ‘analogs’ have recently been discovered at the crossing of the $1\uparrow$ and $2\downarrow$ subbands in high in-plane B [7]. In the region of the analogs, energy levels of opposite spin abruptly rearrange as they populate, forming a completely spin-polarised state. This is thought to be driven by the resulting exchange energy enhancement [20, 22] and resembles the magnetic phase transitions predicted

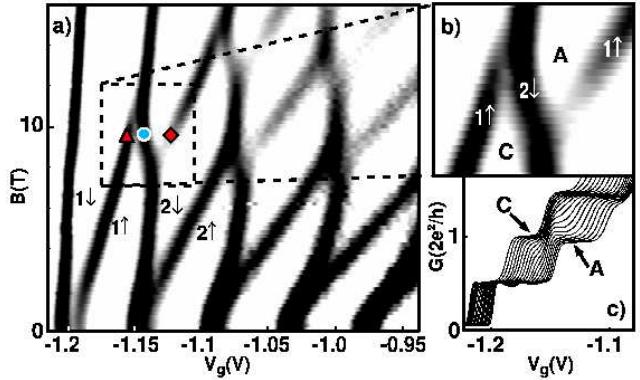


FIG. 1: (Color online) Evolution and crossings of 1D subbands in B . (a) Grey-scale diagram of dG/dV_g as a function of V_g for $B = 0$ to 16 T in increments of 0.2 T. White represents conductance plateaux, and dark lines correspond to a subband populating. Right-moving (left-moving) lines are higher energy spin-up (lower energy spin-down) subbands. We refer to the three symbols later in the paper. (b) A close-up of the $1\uparrow$ and $2\downarrow$ crossing. The trajectory of $1\uparrow$ is discontinuous, with its two parts overlapping in B . **A** and **C** indicate the non-quantized analog and complement structure respectively, shown in (c) Conductance traces from 5.8 T (left) to 13 T, covering the range of figure (b). The analog **A** (a variant of the 0.7 structure) and the complement **C** are indicated.

to occur at crossings of Landau levels [23]. In this paper, we provide direct evidence from DC-bias spectroscopy [24, 25], that the 0.7 structure and analogs are caused by the highest energy spin-up subband pinning to the chemical potential, μ , as predicted by Kristensen and Bruus [10], together with an abrupt rearranging of spin-up and spin-down subbands.

II. SAMPLES AND MEASUREMENT

Our samples consist of split-gate devices defined by electron beam lithography on a Hall bar etched from a high mobility GaAs/Al_{0.33}Ga_{1-0.33}As heterostructure. The two-dimensional electron gas lies 292 nm below the surface of the heterostructure. All the 1DES samples used in this work have a lithographic length of 0.4 μm and a width of 0.7 μm . We used an in-plane B aligned perpendicular to the current direction. We have however observed the same behaviour for in-plane parallel B . By monitoring the Hall voltage, the out-of-plane misalignment was measured to be 0.3°. The measurement temperature was 50 mK.

III. TWO NON-QUANTIZED CONDUCTANCE STRUCTURES RELATED TO REARRANGING OF SUBBANDS AT CROSSINGS

The rearranging of the $1\uparrow$ and $2\downarrow$ subbands in the crossing region is inferred from the data in fig. 1(a), which exhibits multiple crossings of spin-split 1D subbands. The data can be thought of as an energy diagram, where the black lines represent subbands. The close-up of the crossing of $1\uparrow$ and $2\downarrow$ in fig. 1(b) shows that the trajectory of $1\uparrow$ is discontinuous at the crossing with the two parts of $1\uparrow$ overlapping in B . I.e., at around $B = 10$ T, going from left to right in gate-voltage, V_g , $1\uparrow$ populates twice, at two different V_g . Thus, the 1DES energy spectrum is not fixed, but rearranges as the subbands populate, an effect thought to be due to e-e interactions [20, 22, 23].

Two plateau regions, **A** and **C** in fig. 1(b), are formed between the overlapping parts of $1\uparrow$, on the right and left of $2\downarrow$. Although **A** and **C** are non-quantized conductance structures (fig. 1(c)), they are separated by a constant quantized conductance of $\sim 0.5(2e^2/h)$. **A**, the 0.7 analog, has similar properties to the 0.7 structure [4, 7]; in fig. 1(b) the analog region **A** at $B = 10$ T and above is equivalent to the 0.7 structure region near $B = 0$. However, the region below 10 T has no equivalent at $B = 0$ — one cannot investigate $|B| < 0$. In this region (fig. 1(b)), we find a new non-quantized feature, **C**, called a *complement* structure (see fig. 1(c)). As we will show, the DC-bias characteristics of the complement, analog and 0.7 structure provide evidence that they are all caused by pinning of a spin-up subband together with an abrupt drop in energy of a spin-down subband.

IV. BIAS SPECTROSCOPY OF CROSSINGS OF SPIN SUBBANDS IN HIGH MAGNETIC FIELDS

DC-bias (V_{ds}) data taken at 5 T (fig. 2(a)) gives insight into V_{ds} characteristics at the crossing and $B = 0$; the 5 T data is simpler than these regimes, because the spin subbands are far apart in energy. At $V_{\text{ds}} = 0$ in fig. 2(a), each subband gives one dark feature as it intercepts μ . Each

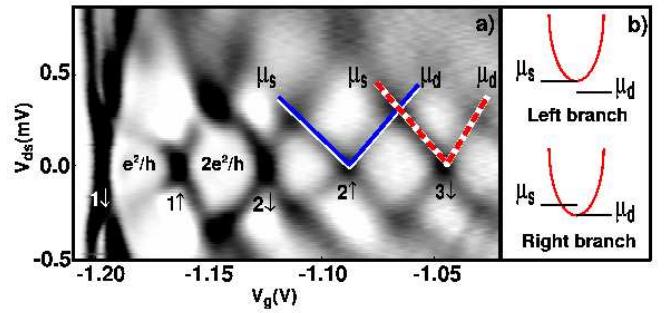


FIG. 2: (Color online) (a) Grey-scale of dG/dV_g data at 5 T as a function of DC-bias V_{ds} and V_g . Labels indicate whether a branch corresponds to a subband intercepting μ_s or μ_d , as shown in (b). A left (right) branch in $+V_{\text{ds}}$ corresponds to a subband intercepting the source (drain) chemical potential $\mu_s = \mu + eV_{\text{ds}}/2$ ($\mu_d = \mu - eV_{\text{ds}}/2$).

of these features splits into a V-shaped pair of branches at $V_{\text{ds}} > 0$ because μ splits in two, μ_s (μ_d) for the source (drain). Left (right) branches are due to subbands intercepting μ_s (μ_d) — see fig. 2(b). The $+V_{\text{ds}}$ branches associated with $2\uparrow$ are marked with a solid blue V-shape and the branches associated with $3\downarrow$ are marked with a dashed red V-shape (n.b. here, and below, the term ‘V-shape’ refers to two branch features in the $+V_{\text{ds}}$ part of the data). In this 5 T data, spin-up features in particular are generally consistent with the non-interacting V_{ds} model [24] in which both branches of the V-shape are present at any B because the subband must pass through both μ_s and μ_d .

In contrast, in the V_{ds} data at $B = 0$ (figs. 3(a) and (b)), there are strong deviations from the expected non-interacting behaviour. Moving from 5 T (fig. 2(a)) to $B = 0$ (fig. 3(a) and (b)), the e^2/h plateau evolves into the 0.7 structure. The V-shaped $1\uparrow$ feature which separates the e^2/h plateau from the $2e^2/h$ plateau moves to the left, forming the grey line marked γ , that separates the 0.7 structure from the $2e^2/h$ plateau. However, γ is no longer a V-shape as it has no left branch — it is just a single right-moving branch (fig. 3(b) and (c)); the expected but ‘missing’ left branch is represented by a dashed line in the schematic diagram in fig. 3(c). γ relates to the $1\uparrow$ subband, so at $B = 0$ although we can detect $1\uparrow$ intercepting μ_d — the right-moving branch from γ — the branch indicating that $1\uparrow$ has intercepted μ_s is missing. We will show that this unexpected behaviour is direct evidence that the 0.7 structure is caused by pinning of the $1\uparrow$ subband as it populates.

Branches are also missing in the V_{ds} data at the crossing (fig. 3(d)), in the region of the analog and complement — the inset to fig. 3(d) shows conductance at the crossing for $V_{\text{ds}} = 0$. Fig. 3(e) gives a schematic of the main features of the crossing [33]. Again, compare fig. 3(d) to the 5 T data in fig. 2(a); the $1\uparrow$ V-shape at 5 T which separates the e^2/h and $2e^2/h$ plateaux has at 9 T shifted

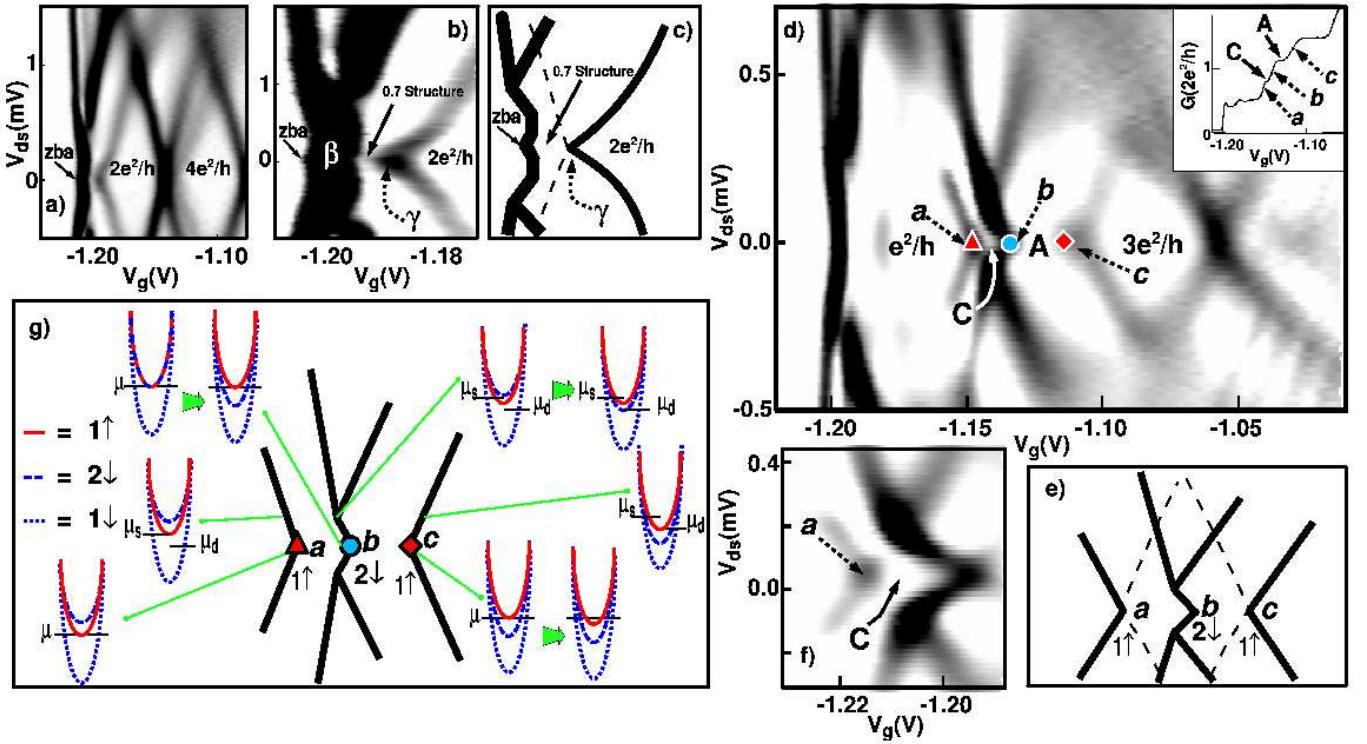


FIG. 3: (Color online) (a) Grey-scale of dG/dV_g at $B = 0$ as a function of V_{ds} . White regions are plateaux. A close-up (b) shows that γ , separating the 0.7 structure from $2e^2/h$, does not split in V_{ds} — the left branch is absent. This is illustrated schematically in (c), where the ‘missing left branches’ are represented by dashed lines. (d) V_{ds} data at the crossing at $B = 9$ T. At $V_{ds} = 0$, **a**, **b** and **c** correspond to the $1\uparrow$, $2\downarrow$ and $1\uparrow$ features marked by symbols in fig. 1. **a** has no right branch in V_{ds} and **c** has no left branch. **b** and **c** beside the analog **A** are equivalent to β and γ beside the 0.7 structure in (b). Inset: Conductance trace for $V_{ds} = 0$ at $B = 9$ T. (e) The ‘missing branches’ for **a** and **c** are represented by dashed lines in this schematic diagram of the crossings. (f) A close-up of **a** and **b**, in data from a similar sample at 8.6 T, demonstrates that **a** has no right branch. (g) A schematic of (d), showing the configurations of subbands. Missing branches indicate that $1\uparrow$ is pinned to μ in the complement and analog regions **A** and **C** (see text).

to the right to form features **a** and **c** in fig. 3(d), whilst $2\downarrow$ causes feature **b**. At $V_{ds} = 0$, points marked with coloured symbols correspond to the symbols in fig. 1(a), and the alternation between $1\uparrow$ (feature **a**), $2\downarrow$ (feature **b**) and $1\uparrow$ (feature **c**) again, indicates that $1\uparrow$ and $2\downarrow$ rearrange as they populate. Feature **a** at the left edge of the complement structure **C** has no right branch in V_{ds} — see the closeup in fig. 3(f), and the schematic in fig. 3(e) in which the absent right branch is represented with a dashed line. Feature **c**, on the right of the analog **A**, has no left branch, and is equivalent to γ in the region of the 0.7 structure in fig. 3(b); feature **b** is equivalent to β . In short, whereas the spin-down feature **b** splits into two branches with increasing V_{ds} , the spin-up features **a** and **c** do not split and only have one branch, either right- or left-moving in V_{ds} . The absence of these branches cannot be understood in a non-interacting electron picture.

A. Missing branches in the bias spectroscopy data indicates ‘pinning’ of spin-up subbands

The missing branches can be explained by a combination of two mechanisms — the abrupt rearranging of the spin-up and spin-down subbands, together with simultaneous pinning of the spin-up subband to μ . In figs. 3(d), (e) and (g), in finite V_{ds} , $1\uparrow$ intercepts μ_s at the left-moving branch of feature **a**, as illustrated by the schematic diagrams of subbands in fig. 3(g). At $V_{ds} = 0$, μ_s is the same as μ , so $1\uparrow$ intercepts μ at **a**. In finite V_{ds} , $1\uparrow$ intercepts μ_d at the right-moving branch from feature **c**. At $V_{ds} = 0$, μ_d is the same as μ , so $1\uparrow$ must still be at μ at feature **c** — $1\uparrow$ reaches μ at **a**, and remains close to μ until feature **c**. In other words, for the left and right branches of $1\uparrow$ (**a** and **c**) to be separated by such a large range in V_g , then at $V_{ds} = 0$, $1\uparrow$ must pin to μ from point **a** until point **c** throughout the regions of the analog and complement. Thus, missing branches on the V-shaped features in the DC-bias data lead directly

to the conclusion that spin-up subbands pin close to the chemical potential over a range of gate-voltages.

B. The contrasting behaviour of spin-up and spin-down subbands, and their rearranging in energy at the crossings

Unlike the $1\uparrow$ subband, $2\downarrow$ does not pin to μ . Also using DC-bias spectroscopy, we have found that spin-down subbands do not give a simple V-shaped feature in V_{ds} [26]. The form of feature **b** for $2\downarrow$ in fig. 3(d), (e) and (g) is typical of spin-down subbands in general. The two branches of the V are not individually resolvable until a certain V_{ds} , here 0.1 mV, has been reached — it is as if the expected V-shaped feature has been ‘collapsed’ along the V_g axis, so the left and right-branches lie on top of each other until $V_{ds} = 0.1$ mV. This implies the exact opposite behaviour for spin-down subbands than for spin-up — it implies that spin-down subbands populate very abruptly, passing through both μ_s and μ_d within a very narrow gate-voltage range, even when μ_s and μ_d are separated in energy by more than 0.1 mV, and in some cases, as much as 0.5 mV [26]. In contrast, for spin-up subbands, it is as if the expected V-shaped feature has been ‘stretched’ along the V_g axis, so the left and right-branches lie far apart from each other in gate-voltage, indicating that spin-up subbands populate very gradually.

Since $2\downarrow$ populates abruptly at **b**, $1\uparrow$ and $2\downarrow$ also rearrange in energy between the complement **C** and analog **A** regions, but with $1\uparrow$ remaining pinned throughout. This rearranging resembles the exchange-driven magnetic phase-transitions predicted for Landau-level crossings [20, 22, 23], and the combination of pinning of spin-up subbands with a sudden drop in energy of spin-down subbands provides an explanation [26] for why the 0.7 structure and analogs, and spin-down features in general [27], remain visible at surprisingly high temperatures.

V. PINNING OF SPIN-UP SUBBANDS IS THE PHENOMENOLOGICAL ORIGIN OF THE NON-QUANTIZED 0.7, COMPLEMENT AND ANALOG STRUCTURES

Pinning of $1\uparrow$ also explains the non-quantized conductances of the complement and analog (fig 4(a)). At $T > 0$, a subband, $N\sigma$, lying close to μ gives a conductance of less than e^2/h , because (ignoring tunnelling and reflection) $G_{N\sigma} = G_0 f(\Delta E, T)$ where $G_0 = e^2/h$ and f is the Fermi function, T is temperature and ΔE is the energy difference between μ and the bottom of the $N\sigma$ subband. If $1\uparrow$ populates only partially at the complement structure and pins close to μ over a range of gate-voltages, then the conductance of this subband, $G_{1\uparrow}$, will be non-quantized and less than e^2/h , i.e., the total conductance of the complement structure $G_{\text{complement}} = G_{1\downarrow} + G_{1\uparrow} = e^2/h + fe^2/h < 2e^2/h$. In earlier work [26], we demon-

strated that in contrast to spin-up subbands, spin-down subbands populate very abruptly and do not pin to μ . We also know from fig. 1(a) that $2\downarrow$ populates between the complement and analog structures. Thus, a quantized increase of $G_{2\downarrow} \sim e^2/h$ is expected when $2\downarrow$ populates, and $G_{\text{analog}} = G_{1\downarrow} + G_{2\downarrow} + G_{1\uparrow} = e^2/h + e^2/h + fe^2/h < 3e^2/h$, i.e., the analog conductance is $\sim e^2/h$ greater than the complement conductance throughout the crossing region, because of the population of $2\downarrow$ between the two structures. Above the analog, $1\uparrow$ goes from being partially to fully populated, giving an increase in G of less than e^2/h , and a total quantized conductance of $3e^2/h$.

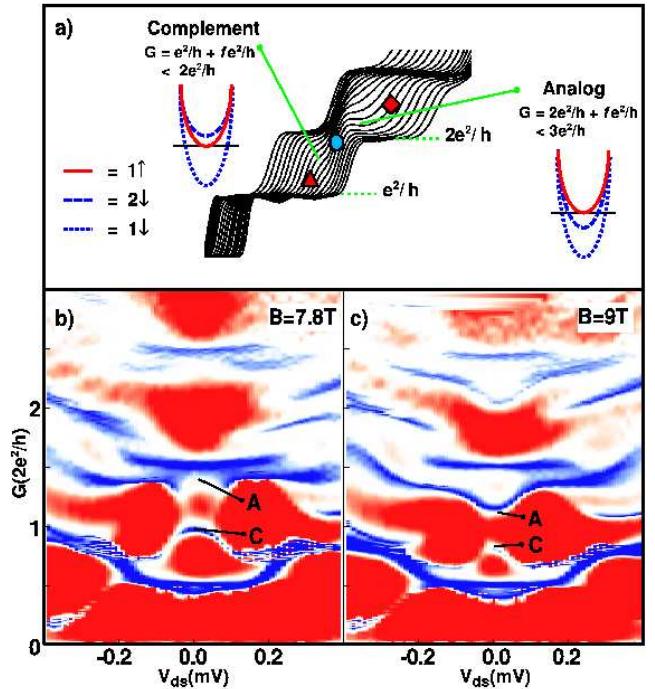


FIG. 4: (Color) (a) Conductance traces from fig. 1(a) with schematics illustrating how pinning of $1\uparrow$ explains the complement and analog. Again, symbols indicate rises in conductance that correspond to features in figs. 1 and 3(d) and (g). (b) dG/dV_g as a function of G and V_{ds} at the crossing at $B = 7.8$ T. Blue indicates plateaux, red indicates abrupt changes in G with V_g and white indicates slowly changing G . (c) Similar data for $B = 9$ T (also the data used to make fig. 3(d)). From 7.8 T to 9 T, the analog, **A**, strengthens as G_{analog} decreases, and the complement, **C**, weakens as $G_{\text{complement}}$ decreases. $G_{\text{complement}}$ and G_{analog} change immediately in finite V_{ds} , unlike the quantized $1.5(2e^2/h)$ plateau which remains at fixed G until it disappears at $V_{ds} \sim \pm 0.2$ mV.

We can perform similar analysis for the 0.7 structure at $B = 0$ by considering $1\downarrow$ and $1\uparrow$ instead of $2\downarrow$ and $1\uparrow$. Near the crossing, $2\downarrow$ and $1\uparrow$ cannot be degenerate when they first populate because we can resolve both features **a** and **b**. However, fig. 3(b) at $B = 0$ has no equivalent to feature **a**, thus $1\downarrow$ and $1\uparrow$ are degenerate when they first populate. Just as the subbands at the crossing

rearrange in energy abruptly, at $B = 0$, the degeneracy between $1\downarrow$ and $1\uparrow$ is abruptly lifted as they populate — $1\downarrow$ drops suddenly in energy [26] to give β , whilst $1\uparrow$ pins to μ between β and γ , giving non-quantized conductance (as first proposed by Kristensen *et al.* [10]), before populating fully at γ . Below the onset of V-shaped splitting from β (for $V_{ds} < 0.3$ mV), there is no left branch from γ because $1\downarrow$ and $1\uparrow$ pass through μ_s together; the missing left branch from $1\uparrow$ is part of feature β , and is separated from the $1\uparrow$ right branch at γ by finite V_g because the subband is pinned.

$G_{\text{complement}}$ and G_{analog} immediately change with V_{ds} (fig. 4(a), (b) and (c)) [34]. The analog **A** at $B = 9$ T rises with increasing V_{ds} from $1.15(2e^2/h)$ to $\sim 1.35(2e^2/h)$, just as the 0.7 structure rises to $\sim 0.85(2e^2/h)$ in finite bias (see refs. [25, 28]), whereas $G_{\text{complement}}$ decreases with increasing V_{ds} . This is in stark contrast to the behaviour of quantized plateaux: for example, the $3e^2/h$ plateau in fig. 4(c) remains at the same G with increasing V_{ds} until it disappears. Quantized plateaux do not change conductance in V_{ds} because, by definition, they occur when the subband edge is some way below μ . Therefore for moderate V_{ds} , the subband still lies well below μ_s and μ_d and G will be unaffected by the energy gap between μ_s and μ_d . The change in $G_{\text{complement}}$ and G_{analog} at small V_{ds} is consistent with $1\uparrow$ pinning close to μ at those features.

VI. DISCUSSION AND CONCLUSIONS

It has been proposed that the rise in G of the 0.7 structure with decreasing T may relate to the Kondo effect [12]. The basis for this theory is the ‘zero-bias anomaly’ (ZBA), a peak in G at $V_{ds} = 0$, similar to that observed in quantum dots [29]. We routinely observe such ZBAs at $B = 0$, which, in a greyscale diagram such as fig. 3(b), take the form of a narrow pointed feature in the pinch-off voltage at $V_{ds} = 0$ (marked **zba** in figs. 3(b) and (c) and indicated by an arrow). However, we have not observed zero-bias anomalies in the conductance at the crossings, despite the presence of the analog structures. Analogs rise in conductance and disappear with decreasing T — if the disappearance of the analog and its large conductance enhancement at low T were due to the Kondo effect, then the enhanced conductance would be destroyed by V_{ds} , and hence, a ZBA would occur. The absence of any ZBA implies that Kondo physics is not the main cause of the enhanced G associated with the 0.7 structure and analog variants at low T .

The phenomenological theory that spin-up subbands pin to μ , but spin-down do not, provides a consistent interpretation for virtually all the characteristics of the 0.7 structure ‘family’. As previously observed [10], pinning of a spin-up subband slightly below μ can explain why the 0.7 structure is typically absent at low T , but appears and decreases in G as T rises — this also applies to

the analogs at crossings. This is not, however, the only T regime associated with the 0.7 structure. In fact, the 0.7 structure only decreases in G with increasing T if it is above $\sim 0.6(2e^2/h)$ at low T — this depends on confining potential, which can be modified by applying a negative voltage to a ‘midline’ gate [30], or by using a scanning probe tip [14]. It was observed that the 0.7 structure sits below $0.6(2e^2/h)$ for negative midline voltages, and certain scanning probe positions, and *rises* in G with increasing T . The same T dependence is observed in an in-plane B field — once the 0.7 structure has moved below $\sim 0.6(2e^2/h)$ due to the B field, it also rises in G with increasing T [9]. In other words, there is a crossover from one T regime to the other, and a low T conductance of $\sim 0.6(2e^2/h)$ marks the crossover. These two T regimes also exist for the analog with a ‘crossover’ conductance of $\sim 1.2(2e^2/h)$ in that case [7]. This second T regime is also consistent with pinning and corresponds to the spin-up subband pinning slightly *above* μ . In addition, at the distinct crossover in B between the two T regimes, G is invariant with T . This is consistent with the subband pinning *exactly at* μ . The rearranging of spin-up and spin-down subbands is also compatible with pinning of spin-up subbands. Taken together, rearranging and pinning explain the presence of *two* non-quantized structures in the crossing region (the complement and analog), their V_{ds} characteristics, and why these non-quantized structures are separated by a quantized conductance.

Additional evidence in support of our interpretation is that similar V_{ds} analysis applied to spin-down subbands explains why the 0.7 structure survives high temperatures [26], and explains other spin-asymmetries [27]. Furthermore, pinning was suggested as an explanation for the unusual thermopower signature of the 0.7 structure [31], and it is also consistent with the 0.7 structure shot-noise [32] signature.

To conclude, we have used DC-bias spectroscopy to study the rearranging of spin-split subbands at crossings. Our results provide *direct* evidence that spin-up subbands pin to μ in the region of the analog and complement structures, and the 0.7 structure. This, combined with the formation of a spin gap [10, 11] and the abrupt drop in energy of the spin-down subband [26] explains the non-quantized conductances of these features, their temperature dependences, and the shot-noise [32] and thermopower [31] signatures of the 0.7 structure. As yet, there is no theory that explains why spin-up subbands should pin in this way at crossings and $B = 0$. We hope that the evidence in this paper will provide the stimulus for theoretical work in this direction.

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- [33] Between taking the fig. 1 data and the figs. 2 and 3 data, the pinch-off voltage changed by $\sim 0.07\text{V}$. We have added 0.07 V to V_g in fig. 1 to aid comparison with the other figures. The change in the device also caused the crossing to shift by -0.5T , so the features in fig. 2 compare better to $B = 9.5\text{T}$ in fig. 1, rather than 9 T.
- [34] A linear correction was added to the data in fig. 4(b) and (c) to allow better resolution of features at both high and low G .